11/10/2023

ERRORS

SIGNIFICANT FIGURES

counting of sig figs :-
1. Measured value
2. Computed/ calculated value

1. Measured Values,

sig figs depends on least count of instrument used to measure that value.

e.g. If ruler of LC = 1 mm is used to measure the length of a rod & it reports ~~19 cm~~ 1900 mm

Last fig ~~0~~ may be erroneous.

Hence 190 0
↑ ← (Doubtful)
Definitely sig figs

→ For measured values,
All known figs (certainly true) + Last doubtful fig = sig figs

Value	# sig figs
1900 mm	3
0.03 cm	1

NOTE: In physics, 190 cm & 1900 mm represent 2 different situations.

190 cm → Doubtful
1900 mm → Doubtful

	190 cm	1900 mm
	sig figs	sig figs
LC of Instrument :	1 cm	1 mm

2. Computed Values

1. All non-zero digits are sig figs.
2. All zeroes b/w non-zero digits are sig figs.
3. Leading zeroes always ~~non sig~~ insignificant.
4. Trailing zeroes only sig figs if decimal pt present.

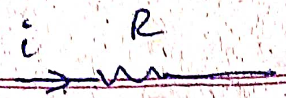
→ Operations on ~~measurements~~ values with sig figs

- Add^n & Sub

$x = 10.02$ cm
$y = 5.1$ cm

$x + y = 10.02 + 5.1 = 15.12 \rightarrow 15.1$ (Report answer with least # decimal places in measurement)

Multiplication & Division



$V = 4.2V$

$i = 0.02A$

Find R.

A. 1. Do regular calculation

$$R = \frac{V}{i} = \frac{4.2}{0.02} = 210$$

2.

Value	sig figs
V	2
i	1
R	2

Round off to least sig. fig.
i.e 1

$\Rightarrow$ $R = 200\ \Omega$

- Round off – (To be done for reporting values after operations) (Second last)

Last digit	Next digit
>5	+1
<5	+0
=5	→ Even → +0
	→ odd → +1

{We are doing mathematical analysis for Random Errors only!} →

ERRORS

→ Systematic Errors

→ Random Errors

- Systematic – Known & can be removed

eg zero error
→ +ve
→ −ve

+ve – Instrument shown +ve value where there should have been a zero

$$\boxed{(\text{Actual value}) = (\text{Measured value}) - (\text{zero error})}$$

$$\boxed{(\text{Zero Correction}) = -(\text{Zero Error})}$$

$$\boxed{(\text{Max. possible error}) = (\text{L.C of instrument})}$$

Random

- ~~Random Errors~~ – Can't be removed or determined their cause

If $A_1, A_2, \ldots, A_n$ are observed values of an exp., we treat the mean of these values as the ~~actual~~ measured value

$$\boxed{A_m = \frac{\left(\sum_{i=1}^{n} A_i\right)}{n}} \leftarrow \left(\text{Mean measured value}\right)$$

Absolute Error in measurement of A_k is $|A_k - A_m|$

$$\boxed{\text{Avg. } \cancel{\text{Error}} \text{ Error } (\Delta A) = \frac{|A_1 - A_m| + |A_2 - A_m| + \dots + |A_n - A_m|}{n}}$$

$$\boxed{\underbrace{A}_{\text{(Actual value)}} = \underbrace{A_m}_{\text{(Measured value)}} \pm \underbrace{\Delta A}_{\text{(Average error)}}}$$

- Fractional error — $\left(\frac{\Delta x}{x}\right)$
- % error — $\left(\frac{\Delta x}{x}\right) \times 100\%$

$z = x + y$

$$z \pm \Delta z = (x \pm \Delta x) + (y \pm \Delta y)$$
$$= (x + y) \pm (\Delta x + \Delta y)$$

$$\Rightarrow \boxed{\Delta z = \Delta x + \Delta y}$$

So, for addn & subn, errors are always added!
(since we are considering max- possible error)

$z = xy$

$$(z \pm \Delta z) = (x \pm \Delta x)(y \pm \Delta y)$$

$$\Rightarrow z \pm \Delta z = xy \pm x\Delta y \pm \Delta x y + \underbrace{(\Delta x \Delta y)}_{\text{neglect!}}$$

$$\Rightarrow \Delta z = (\Delta x)y + x(\Delta y)$$

$$\Rightarrow \frac{\Delta z}{z} = \frac{(\Delta x)y}{z} + \frac{x(\Delta y)}{z}$$

$$\Rightarrow \boxed{\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

OR

$z = xy$

$$\ln(z) = \ln(x) + \ln(y)$$

diff ⇒ $$\boxed{\frac{dz}{z} = \frac{dx}{x} + \frac{dy}{y}}$$

~~for~~ Even if $z = \left(\frac{x}{y}\right)$

$$\boxed{\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}}$$

So, for multiplication & division, fractional errors get added!

NOTE: ① x & y should be independent.

eg. $R = \frac{R_1 R_2}{R_1 + R_2}$

We have to rewrite it as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

& then do error analysis.

② In general, to analyse errors, we use ~~differentiation~~ differentiation

eg. $z = xy^{-2}$ $[(-) \to (+)]$ (Errors should add!)

$\Rightarrow \ln|z| = \ln|x| + 2\ln|y|$

diff

$$\Rightarrow \frac{dz}{z} = \frac{dx}{x} + \frac{2dy}{y}$$

Q. Find error in series & parallel eq. of
$R_1 = 10.0 \pm 0.2\,\Omega$
$R_2 = 20.0 \pm 0.3\,\Omega$

A. For series, $\Delta R = \Delta R_1 + \Delta R_2$

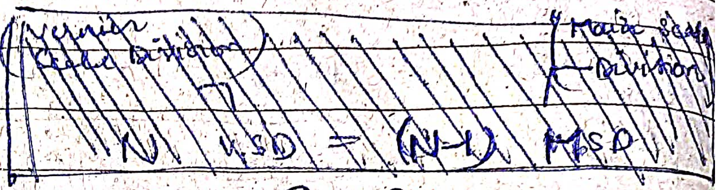
For parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

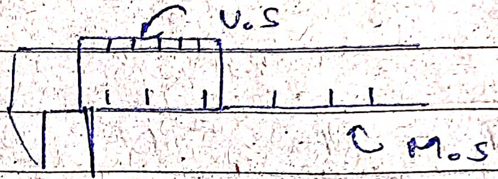
$$\Rightarrow \frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\Rightarrow \Delta R = R^2\left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)$$

INSTRUMENTS

→ Vernier Callipers
→ Main scale (LC = 1 mm)
→ Vernier scale (10 divisions)

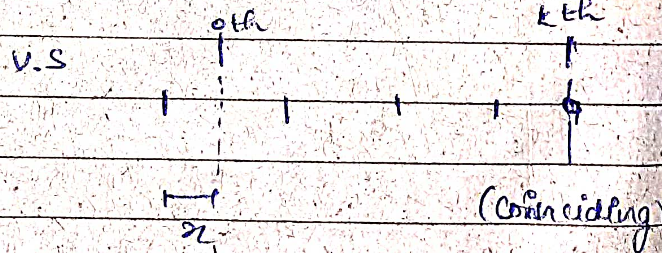




$$\boxed{N \text{ V.SD} = (N-1) \text{ M.SD}}$$

Diff of 1 b/w V.SD & M.SD is necessary for working of a regular vernier calliper.

• Working $(k \le N)$



$$\boxed{x + k \text{ V.S.D} = \left[k \cdot \left(\frac{N-n}{N}\right)\right] \text{M.S.D}}$$

[if N V.SD = (N−n) M.SD]

$$\Rightarrow \boxed{x = 1 - \left\{k\left(\frac{N-n}{N}\right)\right\} \text{M.SD}}$$

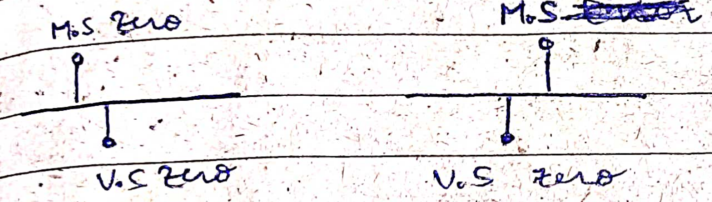
$$\left(\text{Vernier constant}\right) = 1\,M.S.D - 1\,V.S.D$$
$$= \left(1 - \frac{N-1}{N}\right) M.S.D$$
$$= \boxed{\left(\frac{1}{N}\right) M.S.D}$$

$$N\,V.S.D = (N-1)\,M.S.D$$
$$\Rightarrow 1\,V.S.D = \left(\frac{N-1}{N}\right) M.S.D$$

(L.C of Vernier Callipers)

for $n=1$, $\boxed{x = \frac{k}{N}\,M.S.D}$

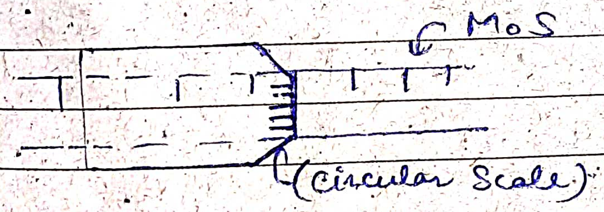
· Zero Error in V.C :



(+ve Zero Error) (−ve Zero Error)

$$\boxed{(\text{Reading}) = M.S.R + x - \left(\text{Zero Error}\right)}$$

→ Screw Gauge



$$\boxed{(\text{Reading}) = M.S.R + \left(\frac{\text{Pitch}}{N}\right)(C.S.R)}$$

N — (# divs. on C.S)

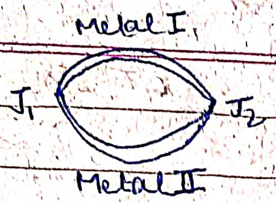
· Zero Error

(+ve) (−ve)

Thermoelectric Effect

1) Seebeck Effect

• Thermocouple =

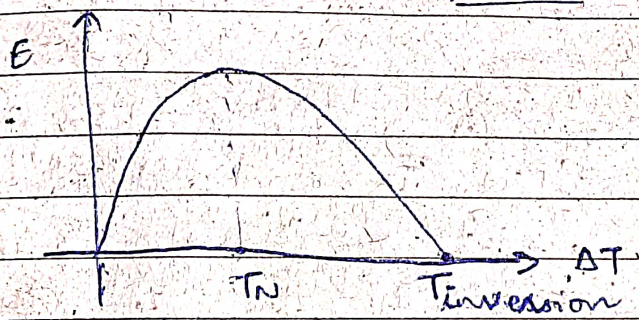


If $T_{J_1} \neq T_{J_2}$

⇓

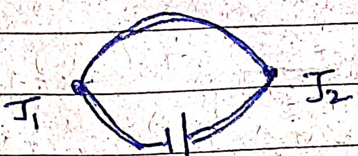
current flows from cold junction to hot junction through Metal which comes ahead in thermoelectrical series.

≡



→ Peltier Effect

If battery inserted in thermocouple, one jn becomes colder & the other hotter.

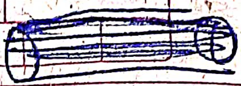


Reverse of Seebeck Effect

→ Thompson Effect

$J_1(T_1)$ $J_2(T_2)$

Conductor

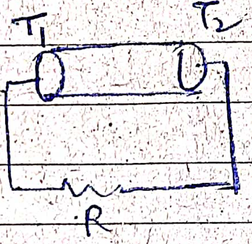


Temp gradient $(T_1 > T_2)$ ⇒ e^- density disturbed

⇓

current from $J_2 \rightarrow J_1$

(e^- from $J_1 \rightarrow J_2$)



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